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CONTRIBUTION TO SOLVING A TWO-DIMENSIONAL CUTTING STOCK PROBLEM WITH TWO OBJECTIVES

Abstract. *In this paper, we propose a technique for solving a two-dimensional cutting stock problem with two-objective. It's about cutting a number of rectangular pieces from a set of raw material plates, themselves identical. These are available in unlimited quantities, where we try to minimize the total area lost and the number of setups to be carried out. This technique is made up of two stages, the first of which consists in generating all the feasible cutting patterns and the second makes it possible to construct cutting plans, satisfying the demands, thanks to a subset of these patterns. These different cutting plans represent all the feasible solutions, each of which is characterized by a number of setups and the total quantity of falls.*

Keywords. *two-dimensional cutting stock problem with two-objective, total area lost, setups, feasible cutting, pattern, cutting plan.*

JEL Classification: C6

1. Introduction

The cutting stock problem, with their different dimensions, to be a real challenge for researchers in operations research. They appear in several production contexts such as fabric, glass, paper, aluminum or cardboard and other metals, etc. In order to improve these problems, several authors have proposed to cut a set of geometric shapes of different dimensions from a cutting pattern with fixed dimensions. In the early 60, (Gilmore and Gomory, 1961), used linear programming methods for solving the one- and two-dimensional cutting stock problem (for an approximate resolution). The same authors (Gilmore and Gomory, 1963; Gilmore and Gomory, 1965) have generalized it to two- and several-dimensional cutting stock problems based on other methods of exact resolution. (Cintra and Wakabayashi, 2004) were interested in the study of the two-dimensional cutting stock problem and packing of bandage when we propose an algorithm based on dynamic programming and column generation, as well as, (Mellouli and Dammak, 2008) who developed an algorithm for two-dimensional

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cutting stock problems based on a procedure for generating cutting patterns. (Stéphane, 2015) have dealt with a two-dimensional cutting stock problem with setups cost in the paper industry by using the genetic algorithm. (Rodrigo et al. 2012) have developed an algorithm, based on the use of the Branch and Bound method to generate the feasible cutting patterns for two-dimensional cutting stock problem, also (Yalaoui and Burkard, 2009) used hybrid optimization by ant colonies for the two-dimensional cutting stock problem. Other more interesting contributions have been proposed by: (Wäscher et al. 2007), (Lodi et al. 2002), (Belov and Scheithauer, 2006), (Michael and Damowicz, 1976), (Suliman Saad, 2001), (Wang, 1983), (Toufik Saadi, 2012), (Zelle and Burkard, 2003).

In this work, we are interested in solving two-dimensional cutting stock problems with two-objective, we seek to minimize the total area lost and the number of setups to be carried out. For this purpose we organize this paper as follows. In the next section, we present the definition and formulation of two-dimensional cutting stock problem with setups. The third section is devoted to the resolution approach. In the fourth section we develop an algorithm to solve the problem. In section five we illustrate the proposed method with examples and we end with a conclusion.

2. Mathematical formulation and methods

Definition 1. We call a feasible cutting pattern a set of pieces cut from an object and the position of each (of them) in the object.

Definition 2. We call a feasible cutting plan a set of feasible cutting patterns making it possible to satisfy the different types of demand.

2.1 Mathematical formulation of the multi-objective optimization problem

A multi-objective combinatorial optimization problem (MOCO) is a decision problem which consists in jointly optimizing a set of k linear objective functions ($k \geq 2$), often conflicting and subject to a set of linear constraints.

This problem is defined by:

$$\begin{array}{l} \text{Optimize } \left(F(x) = f_1(x), f_2(x), \dots, f_k(x)\right) \\ \text{s.t. } \; x \in X. \end{array} \tag{1}$$

Where k is the number of objectives ($k \geq 2$), $x = (x_1, x_2 \dots x_n)$ is the vector representing the decision variables, each of the functions $f_i(x)$ is to be optimized $i = 1 \dots k$, i.e. to say to minimize or maximize and X represents the set of feasible solutions. The set $\mathbb{R}^n$ which contains X is called a decision space. The set $\mathbb{R}^k$ which contains F is called the criteria space or the objective space.

Definition 3. Let be two vectors of criteria $z, z' \in F(X)$. We say that z dominates z' if and only if $z \leq z'$ and $z \neq z'$ (i.e. $z \leq z'$ $\forall$ $i = 1 \dots k$ and $z_i < z'$ for at least one index i). This means that z is at least as good as z' 'in all objectives and, z is strictly better than z' in at least one objective.

Definition 4. A solution $x^* \in X$ is an efficient solution if there is no $x \in X$ such that F (x) dominates F (x^*). Conversely x^* is inefficient.

Therefore, a solution x^* is efficient if its criterion vector is not dominated by any criterion vector of another solution in X. That is, it is not possible to move in a feasible direction to decrease one of the objectives, without necessarily increasing at least one of the other objective values. The limiting efficiency is also known as Pareto optimal and the curve in the objective space formed by the non-dominated vectors which are in the Pareto optimal set is called the Pareto front.

2. 2 Mathematical formulation of the multi-objective cutting problem

We consider a cutting stock problem which consists in cutting a rectangle of length L and width W into several parts called pieces of length $l_i < L$ and width $w_i < W$ where i = 1,…, n, the rectangles in stock are available in unlimited quantities and identical sizes L × W, in which to satisfy orders and minimize the total area lost as well as the number of setups.

In this work we are interested in finding the set of efficient solutions of the following two-objective problem:

$$\begin{aligned}
&\mathrm{Min}\left(f_1(x)\right) = \mathrm{Min}\left(S \times \textstyle\sum_{j=1}^{T} x_j - \sum_{i=1}^{n} s_i d_i\right) \quad \text{(trim loss)}\\
&\mathrm{Min}\left(f_2(x)\right) = \mathrm{Min}\left(\textstyle\sum_{j=1}^{T} \delta(x_j)\right) \quad \text{(stups)}\\
&\textstyle\sum_{j=1}^{T} p_{ij} x_j \geq d_i \qquad i = 1, \dots, n\\
&x_j \in \mathbb{N} \qquad j = 1, \dots, T\\
&\delta(x_j) = \begin{cases} 1 & \text{if } x_j > 0 \\ 0 & \text{if } x_j = 0 \end{cases}
\end{aligned} \tag{2}$$

Where S = L × W is the area of the main leaf, $s_i = l_i \times w_i$ is the area of pieces requested, p_{ij} is the number of occurrences of the i^{th} piece in the j^{th} pattern, d_i is the different types of requests, T is the number of cutting patterns, x_j is the number of times the j^{th} cutting pattern is used, and $\delta(x_j) = \begin{cases} 1 & \text{if } x_j > 0 \\ 0 & \text{if } x_j = 0 \end{cases}$

3. Methods

In this paragraph, we present a technique for solving a two-objective cutting stock problem consisting of two steps: the first consists in generating the achievable cutting patterns. The second step makes it possible to construct cutting plans, satisfying all requests (feasible solutions) and also makes it possible to find the best cutting plans taking into account the two objectives.

3.1 Generation of feasible cutting patterns

The idea of this developed heuristic consists in generating a cutting matrix (denoted P of size m × n), in which each line of P represents a cutting pattern. So according to the classification of all the parts in descending order ($s_1 > s_2 > \ldots > s_n$) and using a non-guillotine cut, the algorithm starts to place the first part c1 of area $s_1 = l_1 \times w_1$ on the rectangle R of the area $S = L \times W$ by $p_{11} = \left\lfloor \frac{S}{s_1} \right\rfloor$, where $\lfloor\rfloor$ is the lower integer part, the following pieces can be placed on the remaining part of S by: for j = 1 and i varying from 2 to n, $p_{1i} = \left\lfloor \frac{S - \sum_{z=1}^{i-1} p_{1z} \times s_z}{s_i} \right\rfloor$, where $s_i = l_i \times w_i$, the elements: $p_{11}, p_{12}, \ldots. p_{1n}$ forms the first row of the matrix P, then the algorithm decreased the number of times the piece c_1 placed on R by 1, ($p_{21} = p_{11}$- l), the following pieces placed on the remaining part of R by: for j = 2 and i varying from 2 to n, $p_{2i} = \left\lfloor \frac{S - \sum_{z=1}^{i-1} p_{2z} \times s_z}{s_i} \right\rfloor$ until the cancellation of p_{11}, The algorithm must be updated by fixing the first part to these values found previously, by i = i + 1, the initialization of h by the number of times where p_{11} is equal to zero, l = 1, j = 2 and for z = 1 to i -1 $p_{h\,z} = p_{j-1,\,z}$ and the second part is varies from $p_{h2} = p_{\,j-1,2}$ - l where l varying from 1 until the cancellation of $p_{j-1,2}$, the following pieces placed on the remaining part of R by: for i varying from 3 to n, $p_{hi} = \left\lfloor \frac{S - \sum_{z=1}^{i-1} p_{hz} \times s_z}{s_i} \right\rfloor$. This process is repeated for each part until i = n-1. The following algorithm explains this situation well:

Algorithm 1

1. Calculate the areas $s_i = l_i \times w_i$, then arrange the s_i in descending order $s_1 > s_2 > s_3, \ldots s_n$, where n is the number of pieces requested.
2. Determine the first row of the matrix P by:
   a) Calculate $p_{11} = \left\lfloor \frac{S}{s_1} \right\rfloor$,
   b) To pose j = 1, for i = 2 to n do $p_{1i} = \left\lfloor \frac{S - \sum_{z=1}^{i-1} p_{1z} \times s_z}{s_i} \right\rfloor$,
3. set i = 1, h = 1, k = 1
   a) set j = 1, d = 1,
   b) if $p_{ji} > 0$ then j = j + 1, l = 1,
   c) h = h + 1,
   d) if i = 1 then $p_{hi} = p_{\,j-1,i}$- l
      for i = k + 1 to n do $p_{hi} = \left\lfloor \frac{S - \sum_{z=1}^{i-1} p_{hz} \times s_z}{s_i} \right\rfloor$,
   e) else for z = 1 to i-1 do $p_{hz} = p_{\,j-1,z}$,
      for i = k do $p_{hi} = p_{\,j-1,i}$- l,
      for i = k + 1 to n do $p_{hi} = \left\lfloor \frac{S - \sum_{z=1}^{i-1} p_{hz} \times s_z}{s_i} \right\rfloor$,
   f) if $p_{hi} > 0$ then set l = l + 1, d = d + 1 and go to (c) else go to (g),
   g) d = d + 1,
   h) if d < m, then go to (b),
   i) else if i < n-1 then set i = i + 1, k = k + 1 and go to (a), else stop,
   j) else set j = j + 1, d = d + 1 and go to (b).

3.2 Construction of the cutting plans

The construction of cutting plans based on the generation of all feasible cutting patterns which makes it possible to construct cutting plans, satisfying the demands, thanks to a subset of these patterns. These different cutting plans represent all the feasible solutions, each of which is characterized by a number of setups and the total quantity of falls, revolves around two stages:

The first consists in grouping together all the cutting patterns which do not deal with the first type of parts and we put it in a set B_1, (practically we search for all the rows that have zeros in the same column of the matrix P and we put it in a set B_1), then we search for all the cutting patterns that do not deal with the second type of parts and we put it in a set B_2, this procedure is carried out until the search for all the cutting patterns which do not process the n^{th} type of parts and we put it in a set B_n.

While the second is justified by the following proposition:

Proposition: Let $\Omega = B_1 \cup B_2 \cup \ldots \cup B_n$ be a subset of $\mathbb{N}$ and $\{P_i\}$ included in at least one subset B_k where $k \geq 1$, if there exists at least one subset $B_d \subset \overline{B_i \cup B_j \cup \ldots \cup B_h}$ (complement of $B_i \cup B_j \cup \ldots \cup B_h$) then $\{P_i\} \cap B_d = \emptyset$.

Prove: by contraposition, we have $P_i\} \cap B_d \neq \emptyset \Rightarrow \forall\ B_d \subset \Omega,\ B_d \not\subset \overline{B_i \cup B_j \cup \ldots \cup B_h}$,

By suppose we have: $\{P_i\}$ included in at least one subset $B_k \Rightarrow \{P_i\} \subset B_k \vee \{P_i\} \subset B_k \wedge B_j \vee \{P_i\} \subset B_k \wedge B_j \ldots \wedge B_h \ldots \vee \{P_i\} \subset B_i \wedge B_j \ldots \wedge B_h \ldots \vee \{P_i\} \subset B_k \wedge B_j \ldots \wedge B_h \ldots \wedge B_n$ so like $\{P_i\} \subset B_i \wedge B_j \ldots \wedge B_h$ then $\{P_i\} \subset B_i \cap B_j \ldots \cap B_h$ and another by $\{P_i\} \cap B_d \neq \emptyset \Rightarrow \{P_i\} \subset B_d \Rightarrow \{P_i\} \subset B_d \cap B_i \cap B_j \ldots \cap B_h \Rightarrow B_d \subset B_i \cup B_j \ldots \cup B_h \Rightarrow B_d \not\subset \overline{B_i \cup B_j \cup \ldots \cup B_h}$.

While section planes are constructed in the following two ways:

1. We arrange the elements of the sets B_k in the non-decreasing order P_1 to P_m where m the number of the elements of all the sets then for i varying from 1 to m-1 we determine P_i belong to sets B_k, where k = 1, 2 … and subsets of (l -1) elements in which P_i and the (l -1) elements do not belong to the same set as well P_i and the (l -2) elements do not form a cutting plane in previous step where l = 2, 3 ...
2. We add to the best cutting plans on the two objective functions of the previous step a cutting pattern in order to reduce the different types of demand.

The construction of the cutting planes is continuous, until either $\sum_{j=1}^{T} X_j$ theoretical value $\sum_{j=1}^{T} X_j = \left\lceil \frac{\sum_{i=1}^{n} s_i \times d_i}{S} \right\rceil$ or is no improvement of the solutions.

Algorithm 2

1. Calculate the theoretical value: $\sum_{j=1}^{T} X_j = \left\lceil \frac{\sum_{i=1}^{n} s_i \times d_i}{S} \right\rceil$, where $\lceil\ \rceil$ is the upper whole part
2. If there is a line $P_i \in P$, does not contain zeros, then determine the cutting plane $pd_k = P_i$, where i, k are positive integers and go to (a), else go to (3),

a) Calculate $x_j = \text{Max}\left(\left\lceil \frac{d_i}{P_{ij}} \right\rceil\right)$, where $P_{ij} \neq 0$,
b) Calculate the trim loss: $pt_i = S \times x_j - \sum_{r=1}^{n} s_r\, d_r$,
c) Calculate $Pt = \text{Min}\ (Pt_i)$,
3. For j = 1 to n do
For i = 1 to m do
a) If $P_{ij} = 0$, then $A_{ij} = \{P_i\}$, else $A_{ij} = \emptyset$,
b) For i = 1 to m do $B_j = \cup_{i=1}^{m} A_{ij}$,
4. Arrange the cutting patterns in non-decreasing order P_1 to P_m,
5. To pose l = 2, t =1,
a) If there are cutting plans with l - 1 number of setups, then add to each effective plan at l - 1 number of setups, a cutting pattern in such a way to reduce the largest values of $\left\lceil \frac{d_i}{h_{ij}} \right\rceil$ where $b_{ij} = \sum_{i\geq 1} P_{ij}$ and go to (b) else go to (b),
b) For i = t to m-1 do
1. Determine P_i belong to sets B_k, k = 1, 2, ….
2. If there are subsets of (l -1) elements in which P_i and the (l -1) elements do not belong to the same set thus the (l - 2) elements do not form cutting planes in the previous step then determine the cutting planes $Pd_h = \begin{cases} P_i \\ (l-1) \text{ elements} \end{cases}$, h = 1, 2…, and to pose t = t + 1 else to pose t = t + 1 and go to (b)
c) Calculate $h_{ij} = \sum_{i\geq 1} P_{ij}$, $x_j = \text{Max}\left(\left\lceil \frac{d_i}{h_{ij}} \right\rceil\right)$, $x_{pd_i} = \sum_{j=1}^{T} x_j$, and Min ($x_{pd_i}$),
d) If there are x_{pd_i} redundant are equal and $x_{pd_i} >$ Min (x_{pd_i})
Then eliminate redundant x_{pd_i} and go to (e) else go to (e),
e) Calculate the trim loss: $pt = S \times \text{Min}\left(x_{pd_i}\right) - \sum_{r=1}^{n} s_r d_r$,
f) If $\sum_{j=1}^{T} X_j$ reaches or is no improvement of the solutions, stop, else set l: = l + 1 and go to (a).

3.3 Fineness of the Algorithm

We check in this paragraph that the number of iterations is finite and the algorithm does not loop:

Cutting pattern generation algorithm: The idea in this step consists in calculating the frequency p_{11} of the first part s_1 on the main rectangle S and each time decreased this frequency by 1 until the cancellation of p_{11} and calculate the other frequencies by $p_{1i} = \left\lfloor \frac{S - \sum_{z=1}^{i-1} p_{1z} \times s_z}{s_i} \right\rfloor$, the algorithm reiterated for each part until i = n-1. Indeed a considered part is not revisited a second time, so the algorithm does not loop and as the number of parts is finite then the number of iterations are finite,

The proposed algorithm consists of searching for one by the lines that contain zeros from the first column, until the rows that contain zeros from the n^{th} column, in a matrix of size m × n, so the algorithm starts from the first column and arrives at the n^{th} column, therefore, the algorithm does not loop and as the matrix is of finite size then the number of iterations is finite, the other by in the sets B_j which to build from the first step, the algorithm searches in each iteration for the subsets of l elements in which each l-1 where l = 2, 3…, elements belong to different sets, up to which the stop test is reached, therefore, the algorithm is finished and does not loop.

4. Results and discussion

4.1. Results

We present in this section some results obtained by our proposed approach, as well as a comparative study with methods existing in the literature.

Example 1. In this example we run the proposed algorithm

- $S = L \times W = 60 \times 40 = 2400$
- $n = 3$,
- $s_i = (l_1 \times w_1, l_2 \times w_2, l_3 \times w_3) = (50 \times 25, 25 \times 20, 20 \times 12) = (1250, 500, 400)$
- $d = (d_1, d_2, d_3) = ((2, 3, 1)$.

This example presented explaining the flow of our proposed algorithm.

1. Arrange in descending order, 1250 > 500 > 400,
2. Determine the first row of the matrix P by: $p_{11} = \left\lfloor \frac{2400}{1250} \right\rfloor = 1$
   a) To pose j = 1, for i = 2 to 3 do $p_{12} = \left\lfloor \frac{2400 - 1250 \times 1}{500} \right\rfloor = 2$, $P_{13} = \left\lfloor \frac{2400 - 1250 \times 1 - 500 \times 2}{400} \right\rfloor = 0$, $P = (1 \quad 2 \quad 0)$
3. To pose i = 1, h = 1, k = 1,
   a) To pose j =1, d =1,
   b) $P_{11} = 1 > 0$ then j = j + 1 = 2, l = 1,
   c) h := h + 1= 2,
   d) i = 1, $p_{21} = p_{11} - 1 = 0$,
   For i = 2 to 3 do $p_{22} = \left\lfloor \frac{2400 - 1250 \times 0}{500} \right\rfloor = 4$, $p_{23} = \left\lfloor \frac{4000 - 1250 \times 2 - 3 \times 500}{400} \right\rfloor = 0$,
   $P = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 4 & 0 \end{pmatrix}$,
   e) $P_{21} = 0$, go to (g),
   g) d = d +1= 2,
   h) i = 1< n -1 = 1 then i = i + 1= 2, k = k + 1 = 1 + 1= 2 and go to (a),
   a) To pose j = 1, d = 1,
   b) $P_{12} = 2, > 0$ then j = j + 1 = 2, l = 1,
   c) h = h + 1 = 3
   d) i > 1 then for z =1 to 1 do $p_{31} = p_{11} = 1$,
   for i = k = 2 do $p_{32} = p_{12} - 1 = 2 - 1 = 1$,
   for i = 3 to 3 do $p_{33} = \left\lfloor \frac{2400 - 1250 \times 1 - 1 \times 500}{400} \right\rfloor = 1$,

   $P = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 4 & 0 \\ 1 & 1 & 1 \end{pmatrix}$

We continue the generation the achievable cutting patterns, the results in the following matrix

Table 1. Matrix of feasible cutting patterns

$$P = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 4 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \\ 0 & 3 & 2 \\ 0 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 6 \end{pmatrix}$$

1. $\sum_{j=1}^{T} X_j = \left\lceil \frac{\sum_{i=1}^{n} s_i \times d_i}{S} \right\rceil = \left\lceil \frac{4400}{2400} \right\rceil = 2$,
2. $Pd_1 = (1, 1, 1)$ is a cutting plane, $X_{pd_1} = \text{Max}\,(\frac{2}{1}, \frac{3}{1}, \frac{1}{1}) = 3$
   $Pt_1 = 2400 \times 3 - (1250 \times 2 + 500 \times 3 + 400 \times 1) = 28\%$,
3. **1th iteration:** For j = 1, for i = 1 to 8 do
   a) $P_{11} = 1$, then $A_{11} = \emptyset$, $P_{21} = 0$, then $A_{21} = \{P_2\}$, $P_{31} = 1$, then $A_{31} = \emptyset$, $P_{41} = 1$, then $A_{41} = \emptyset$, $P_{51} = 0$, then $A_{51} = \{P_5\}$, $P_{61} = 0$, then $A_{61} = \{P_6\}$, $P_{71} = 0$, then $A_{71} = \{P_7\}$, $P_{81} = 0$, then $A_{81} = \{P_8\}$, go to (b),
   b) For i = 8 to pose $B_1 = \bigcup_{i=1}^{8} A_{i1}$, $= A_{11} \cup A_{21} \cup A_{31} \cup A_{41} \cup A_{51} \cup A_{61} \cup A_{71} \cup A_{81} = \emptyset \cup \{P_2\} \cup \emptyset \cup \emptyset \cup \{P_5\} \cup \{P_6\} \cup \{P_7\} \cup \{P_8\} = \{P_2, P_5, P_6, P_7, P_8\}$,

2th iteration: For j = 2, for i = 1 to 8 do
   a) $P_{12} = 2$, then $A_{12} = \emptyset$, $P_{22} = 4$, then $A_{22} = \emptyset$, $P_{32} = 1$, then $A_{32} = \emptyset$, $P_{42} = 0$, then $A_{42} = \{P_4\}$, $P_{52} = 3$, then $A_{52} = \emptyset$, $P_{62} = 2$, then $A_{62} = \emptyset$, $P_{72} = 1$, then $A_{72} = \emptyset$, $P_{82} = 0$, then $A_{82} = \{P_8\}$and go to (b),
   b) For i = 8 to pose $B_2 = \bigcup_{i=1}^{8} A_{i2}$, $= A_{12} \cup A_{22} \cup A_{32} \cup A_{42} \cup A_{52} \cup A_{62} \cup A_{72} \cup A_{82} = \emptyset \cup \emptyset \cup \{P_4\} \cup \emptyset \cup \emptyset \cup \emptyset \cup \emptyset \cup \{P_8\} = \{P_4, P_8\}$,

3th iteration: For j: = 3, for i = 1 to 8 do
   a) $P_{13} = 0$, then $A_{13} = \{P_1\}$, $P_{23} = 0$, then $A_{23} = \{P_2\}$, $P_{33} = 1$, then $A_{33} = \emptyset$, $P_{43} = 2$, then $A_{43} = \emptyset$, $P_{53} = 2$, then $A_{53} = \emptyset$, $P_{63} = 3$, then $A_{63} = \emptyset$, $P_{73} = 4$, then $A_{73} = \emptyset$, $P_{83} = 6$, then $A_{83} = \emptyset$ go to (b),
   b) For i = 8, to pose $B_3 = \bigcup_{i=1}^{8} A_{i3}$, $= A_{13} \cup A_{23} \cup A_{33} \cup A_{43} \cup A_{53} \cup A_{63} \cup A_{73} \cup A_{83} = \{P_1\} \cup \{P_2\} \cup \emptyset \cup \emptyset \cup \emptyset \cup \emptyset \cup \emptyset \cup \emptyset = \{P_1, P_2\}$,

We have: $B_1 = \{P_2, P_5, P_6, P_7, P_8\}$, $B_2 = \{P_4, P_7\}$, $B_3 = \{P_1, P_2\}$,

1th iteration : to pose l = 2,

$\exists\, Pd_1 = (1, 1, 1)$ cutting plans with 1 number of setups, we add to Pd_1, P_1 in such a way to reduce $\left\lceil \frac{d_1}{P_{11}} \right\rceil$ and $\left\lceil \frac{d_2}{P_{22}} \right\rceil$. So $Pd_2 = (P_1, P_4)$ a cutting plane with 2 numbers of setups.

$Pd_2 =$

$\begin{cases} P_1 = (1,2,0) \\ P_3 = (1,1,1) \end{cases}$, we calculate: $x_{pd_2} = \begin{cases} \text{Max}\left[\frac{2}{2}, \frac{3}{3}, 0\right] = 1 \\ \text{Max}\left[\frac{2}{2}, \frac{3}{3}, \frac{1}{1}\right] = 1 \end{cases}$

a) Arrange the cutting patterns in non-decreasing order P_1 to P_8
b) For i = 1 to 8 do
1. determine P_i belongs to sets B_k, where k = 1, 2, 3
2. For $P_1 \in B_3$, $\exists$ $\{P_4\},\{P_5\},\{P_6\}$, $\{P_7\}$, $\{P_8\}$ three subsets, in which P_4, P_5, P_6, P_7 and P_8, does not belong to B_3, thus the P_4, P_5, P_6, P_7 the same P_8 do not form cutting planes in the previous step then
   $Pd_3 = \begin{cases} P_1 \\ P_4 \end{cases}$, $Pd_4 = \begin{cases} P_1 \\ P_5 \end{cases}$, $Pd_5 = \begin{cases} P_1 \\ P_6 \end{cases}$, $Pd_6 = \begin{cases} P_1 \\ P_7 \end{cases}$, $Pd_7 = \begin{cases} P_1 \\ P_8 \end{cases}$,
   calculate: $x_{pd_3} = \begin{cases} \text{Max}\left[\frac{2}{2}, \frac{3}{2}, 0\right] = 2 \\ \text{Max}\left[\frac{2}{2}, 0, \frac{1}{2}\right] = 1 \end{cases}$, $x_{pd_4} = \begin{cases} \text{Max}\left[\frac{2}{1}, \frac{3}{5}, 0\right] = 2 \\ \text{Max}\left[0, \frac{3}{5}, \frac{1}{2}\right] = 1 \end{cases}$ x_{pd_6},
   x_{pd_7},

We represent all the feasible solutions with two numbers of setups in the following table:

Table 2. Set of feasible solutions for two numbers of installations

N° Solution	Feasible solutions	The number of times the pattern j is used (x_j)	Percent (%) of trim loss	Number of setups
1	$P_1 = (1, 2, 0)$ $P_3 = (1, 1, 1)$	$\begin{cases} 1 \\ 1 \end{cases}$	4.00	2
2	$P_1 = (1, 2, 0)$ $P_4 = (1, 0, 2)$	$\begin{cases} 2 \\ 1 \end{cases}$	28.00	2
3	$P_4 = (1, 0, 2)$ $P_6 = (0, 2, 3)$	$\begin{cases} 2 \\ 2 \end{cases}$	52.00	2
4	$P_4 = (1, 0, 2)$ $P_7 = (0, 1, 4)$	$\begin{cases} 2 \\ 3 \end{cases}$	76.00	2

$\text{Min}(x_{pd}) = \text{Min}(x_{pd_2}, x_{pd_3}, x_{pd_4}, x_{pd_5}, x_{pd_6}, x_{pd_7}, x_{pd_8}, x_{pd_9}, x_{pd_{10}}) = x_{pd_2}$.

Stop, because the theoretical value of $\left\lceil \sum_{j=1}^{T} X_j \right\rceil = \left\lceil \frac{4400}{2400} \right\rceil = 2$, so $\mathrm{Min}(x_{pd}) = x_{pd_2} = 2$,

c) Pt = 2400 × (1 + 1) - 4400 = 4 %,

So all the effective solutions presented in the following table:

Table 3. Set of effective solutions

N^0 of Solutions	Effective solutions	Percent (%) of trim loss	Number of setups
1	Pd_2	4.00	2
2	Pd_1	28.00	1

Example 2. We use in this section, an example cited in [7]. A floor tile manufacturing plant uses rectangular shaped marble sheets of length 3000 mm and width 1400 mm as raw material to cut tiles according to the given specifications.
The company has received an order for kitchen tiles according to the dimensions given below:
• S = 3000 x 1400 = 4×10^4,
• n = 6,
• s = (l_1 x w_1, l_2 x w_2, l_3 x w_3, l_4 x w_4, l_5 x w_5, l_6 x w_6) = (132 x10^4, 80 x10^4, 112 x10^4, 84x10^4, 60x10^4, 128 x10^4),
• d = (d_1, d_2, d_3, d_4, d_5, d_6) = (1, 5, 4, 1, 1, 3).

a. Algorithm 1 is applied to generate the feasible cutting patterns the results are illustrated in the annex1 table.
b. $\sum_{j=1}^{T} X_j = \left\lceil \frac{\sum_{i=1}^{n} s_i \times d_i}{S} \right\rceil = \left\lceil \frac{1508\times10^4}{420\times10^4} \right\rceil = 4$,
c. There is not a cutting pattern applied to all types of parts, so go to (d),
d. B_1 = {P_4, P_8, P_9, P_{10}, P_{11}, P_{12}, P_{13}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25}, P_{26}, P_{27}, P_{28}, P_{36}, P_{37}, P_{38}, P_{39}, P_{40}, P_{41}, P_{42}, P_{43}, P_{44}, P_{45}, P_{46}, P_{47}, P_{48}, P_{49}, P_{50}, P_{51}, P_{52}, P_{53}, P_{54}, P_{55}, P_{56}, P_{57}, P_{58}, P_{59}, P_{60}, P_{73}, P_{74}, P_{75}, P_{76}, P_{77}, P_{78}, P_{79}, P_{80}, P_{81}, P_{82}, P_{83}, P_{84}},
B_2 = {P_1, P_5, P_7, P_{10}, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25} P_{26}, P_{27}, P_{28}, P_{29}, P_{31}, P_{32}, P_{33}, P_{34}, P_{35}, P_{43}, P_{44}, P_{45}, P_{46}, P_{47}, P_{48}, P_{49}, P_{50}, P_{51}, P_{52}, P_{53}, P_{54}, P_{55}, P_{56}, P_{57}, P_{58}, P_{59}, P_{60}, P_{61}, P_{62}, P_{63}, P_{64}, P_{65}, P_{66}, P_{67}, P_{68}, P_{69}, P_{70}, P_{71}, P_{72}, P_{80}, P_{81}, P_{84}},
B_3 = { P_1, P_2, P_3, P_4, P_{13}, P_{14}, P_{15}, P_{17}, P_{18}, P_{19}, P_{25}, P_{26}, P_{27}, P_{28}, P_{29}, P_{30}, P_{33}, P_{34}, P_{35}, P_{36}, P_{39}, P_{40}, P_{41}, P_{42}, P_{47}, P_{48}, P_{49}, P_{50}, P_{51}, P_{52}, P_{53}, P_{54}, P_{55}, P_{56}, P_{57}, P_{58}, P_{59}, P_{60}, P_{61}, P_{63}, P_{67}, P_{68}, P_{69}, P_{70}, P_{71}, P_{72}, P_{73}, P_{77}, P_{78}, P_{79}, P_{80}, P_{81}, P_{82}},
B_4 = { P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{21}, P_{24}, P_{28}, P_{29}, P_{30}, P_{32}, P_{35}, P_{36}, P_{38}, P_{41}, P_{43}, P_{45}, P_{46}, P_{56}, P_{57}, P_{58}, P_{59}, P_{60}, P_{61}, P_{62}, P_{63}, P_{65}, P_{66}, P_{67}, P_{68}, P_{69}, P_{73}, P_{75}, P_{76}, P_{77}, P_{78}, P_{79}, P_{82}, P_{84}},

B_5 = { P_1, P_2, P_3, P_4, P_5, P_6, P_8, P_{10}, P_{11}, P_{12}, P_{13}, P_{16}, P_{17}, P_{19}, P_{42}, P_{43}, P_{44}, P_{46}, P_{47}, P_{49}, P_{51}, P_{55}, P_{60}, P_{61}, P_{63}, P_{64}, P_{66}, P_{69}, P_{70}, P_{72}, P_{73}, P_{74}, P_{76}, P_{79}, P_{83}}

B_6 = { P_1, P_2, P_3, P_4, P_5, P_6, P_8, P_{10}, P_{11}, P_{12}, P_{13}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{25}, P_{26}, P_{27}, P_{28}, P_{30}, P_{31}, P_{32}, P_{33}, P_{34}, P_{35}, P_{36}, P_{37}, P_{38}, P_{39}, P_{40}, P_{41}, P_{81}, P_{83}, P_{84}},

1th iteration: To pose l = 2,

a) ∄ cutting plans with 1 number of setups, so go to (b),
b) Arrange the cutting patterns in non-decreasing order P_1 to P_{84}
c) For i = 1 to 83 do

1. Let $P_1 \in B_2$, $P_1 \in B_3$, $P_1 \in B_4$, $P_1 \in B_5$, $P_1 \in B_6$,
2. ∄ Subsets of (2 -1) elements in which P_1 and the (2 -1) elements do not belong to the same, set t = t + 1 = 1 + 1 = 2 and go to (b).

1. Let $P_2 \in B_3$, $P_2 \in B_4$, $P_2 \in B_5$, $P_2 \in B_6$,
2. ∃ {P_{22}},{P_{23}} three subsets, in which P_{22} and P_{23}, does not belong to B_3, B_4 B_5 and B_6 thus the P_{22}, P_{23} do not form cutting planes in the previous step then $Pd_1 = \begin{cases} P_2 = (2\ 1\ 0\ 0\ 0\ 0) \\ P_{22} = (0\ 0\ 1\ 2\ 1\ 1) \end{cases}$, $Pd_1 = \begin{cases} P_2 = (2\ 1\ 0\ 0\ 0\ 0) \\ P_{23} = (0\ 0\ 1\ 1\ 2\ 1) \end{cases}$. We continue in the same way and we eliminate the redundant plans.

We represent the results in the following table:

Table 4. Feasible solutions for two numbers of setups

N°	Feasible solutions	The number of times the pattern j is used (x_j)	Percent (%) of trim loss $\times 10^4$	Number of setups
1	P_{16} = (1, 0, 1, 2, 0, 0) P_{62} = (1, 1, 0, 0, 1, 1)	$\begin{cases} 4 \\ 5 \end{cases}$	22.72	2
2	P_3 = (1, 2, 0, 0, 0, 0) P_{22}= (0, 0, 1, 2, 1, 1)	$\begin{cases} 3 \\ 4 \end{cases}$	12.56	2
3	P_6 = (1, 1, 1, 0, 0, 0) P_{22}= (0, 0, 0, 3, 1, 1)	$\begin{cases} 3 \\ 5 \end{cases}$	18.52	2
4	P_{17} = (1, 0, 0, 3, 0, 0) P_{22}= (0, 1, 1, 0, 1, 1)	$\begin{cases} 5 \\ 1 \end{cases}$	10.12	2
5	P_7 = (1, 0, 2, 0, 0, 1) P_{18}= (0, 2, 0, 1, 1, 0)	$\begin{cases} 3 \\ 3 \end{cases}$	10.12	2

6	$P_8 = (0, 2, 1, 0, 0, 0)$ $P_{71} = (1, 0, 0, 1, 1, 2)$	$\begin{cases} 4 \\ 2 \end{cases}$	10.12	2
7	$P_9 = (0, 1, 2, 0, 0, 1)$ $P_{33} = (1, 0, 0, 2, 1, 0)$	$\begin{cases} 5 \\ 1 \end{cases}$	10.12	2

$Min(Pt) = 420 \times 10^4 \times 6 - 1508 \times 10^4 = 1012 \times 10^4$, this result corresponds to the solution: Pd_4, Pd_5, Pd_6, Pd_7, so this solution is effective.

$\sum_{j=1}^{T} X_j = 6$ (For effective solutions) greater than the theoretical value $\sum_{j=1}^{T} X_j = 4$. So go to new iteration.

2^{th} iteration:

a) $\exists$ $Pd_6 = (P_8, P_{71})$ effective solution with 2 number of setups,

We add to $Pd_6 = (P_8, P_{71})$, P_{43} in such a way to reduce $\left\lceil \frac{d_3}{P_{33}} \right\rceil$, so $Pd_8 = (P_8, P_{43}, P_{71})$ a cutting plane with 3 numbers of setups. Go to (b),

b. We eliminate the redundant plans then we represent the results in the following table:

Table 5. Feasible solutions for two numbers of setups

N°	Feasible solutions	The number of times the pattern j is used (x_j)	Percent (%) of trim loss $\times 10^4$	Number of setups
8	$P_8 = (0, 2, 1, 0, 0, 0)$ $P_{16} = (1, 0, 1, 2, 0, 0)$ $P_{62} = (1, 1, 0, 0, 1, 1)$	$\begin{cases} 2 \\ 2 \\ 3 \end{cases}$	12.56	3
9	$P_3 = (1, 2, 0, 0, 0, 0)$ $P_{22} = (0, 0, 1, 2, 1, 1)$ $P_{76} = (0, 1, 1, 0, 0, 3)$	$\begin{cases} 2 \\ 2 \\ 1 \end{cases}$	5.92	3
10	$P_9 = (0, 1, 2, 0, 0, 1)$ $P_{31} = (1, 0, 1, 1, 1, 0)$ $P_{76} = (0, 1, 1, 0, 0, 3)$	$\begin{cases} 3 \\ 1 \\ 1 \end{cases}$	5.92	3
11	$P_8 = (0, 2, 1, 0, 0, 0)$ $P_{43} = (0, 0, 3, 0, 0, 1)$ $P_{71} = (1, 0, 0, 1, 1, 2)$	$\begin{cases} 3 \\ 2 \\ 1 \end{cases}$	10.12	3

3^{th} iteration:

$Min(Pt) = 420 \times 10^4 \times 5 - 1508 \times 10^4 = 592 \times 10^4$, this result corresponds to the solution: Pd_9, Pd_{10} so this solution is effective.

$\sum_{j=1}^{T} X_j = 5$ (For effective solutions) greater than the theoretical value $\sum_{j=1}^{T} X_j = 4$.
We eliminate the redundant plans then we represent the results in the following table:

Table 6. Feasible solutions for two numbers of setups

N°	Feasible solutions	The number of times the pattern j is used (x_j)	Percent (%) of trim loss $\times 10^4$	Number of setups
12	$P_8 = (0, 2, 1, 0, 0, 0)$ $P_{16} = (1, 0, 1, 2, 0, 0)$ $P_{62} = (1, 1, 0, 0, 1, 1)$ $P_{76} = (0, 1, 1, 0, 0, 3)$	2 2 2 1	12.56	4
13	$P_3 = (1, 2, 0, 0, 0, 0)$ $P_8 = (0, 1, 2, 0, 0, 0)$ $P_{22} = (0, 0, 1, 2, 1, 1)$ $P_{76} = (0, 1, 1, 0, 0, 3)$	2 2 1 1	10.12	4
14	$P_4 = (0, 3, 0, 0, 0, 0)$ $P_8 = (0, 2, 1, 0, 0, 0)$ $P_{43} = (0, 0, 3, 0, 0, 1)$ $P_{71} = (1, 0, 0, 1, 1, 2)$	1 1 1 1	1.72	4

$\mathrm{Min}(Pt) = 420 \times 10^4 \times 4 - 1508 \times 10^4 = 172 \times 10^4$, this result corresponds to the solution: Pd_{14}, so this solution is effective.
$\sum_{j=1}^{T} X_j = 4$ (For effective solutions) greater than the theoretical value $\sum_{j=1}^{T} X_j = 4$.
So stop.
So the effective solutions are presented in the following table:

Table 7. Effective solutions.

N° of solution	Solutions efficaces	Percent (%) of trim loss $\times 10^4$	Number of setups
1	$P_4 = (0, 3, 0, 0, 0, 0)$ $P_8 = (0, 2, 1, 0, 0, 0)$ $P_{43} = (0, 0, 3, 0, 0, 1)$ $P_{71} = (1, 0, 0, 1, 1, 2)$	1.72	4

2	$P_8 = (0, 2, 1, 0, 0, 0)$ $P_{43} = (0, 0, 3, 0, 0, 1)$ $P_{71} = (1, 0, 0, 1, 1, 2)$	5.92	3
3	$P_8 = (0, 2, 1, 0, 0, 0)$ $P_{71} = (1, 0, 0, 1, 1, 2)$	10,12	2

Example 3.

- $S = 6480 \times 342020$,
- $n = 13$,
- $s = (s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9, s_{10}, s_{11}, s_{12}, s_{13}) = (510\times 250, 600 \times 235, 680 \times 186, 720 \times 376, 730 \times 220, 760 \times 224, 900\times 410, 950\times 400, 1020 \times 520, 1100\times 632, 1140\times 547, 1200 \times 643, 1356\times 700)$,
- $d = (d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8, d_9, d_{10}, d_{11}, d_{12}, d_{13}) = (54, 18, 61, 17, 33, 14, 12, 12, 32, 30, 22, 32, 54)$.

Table 8.Result obtained by our algorithm

N° effective solutions	Percent (%) of Trims loss	Number of setups
1	1.41	11
2	1.83	10
3	2.07	9
4	3.61	8
5	3.79	7
6	4.03	6
7	5.12	5
8	6.22	4

4.2 Discussion

In this empirical study, we observed that the multi-objective method is useful because it provides a set of effective solutions which are used to give the choice to the decision maker.

5. Conclusion

In this work, we have adopted a multi-objective approach, to solve a two-dimensional cutting stock problem with a setup cost, the problem, therefore, consists in minimizing two objective functions: the total trims loss of the raw material and the number of setups, under the constraint of fulfilling the order. This approach makes it possible to obtain a set of efficient solutions which are not generally evaluated by a common scalar function.

The technique is abundantly efficient in a wide range of examples, when taking into account the result obtained by other methods exists in the literature.

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